



**Oliver Schuetze**



Álgebra • Oliver Schuetze

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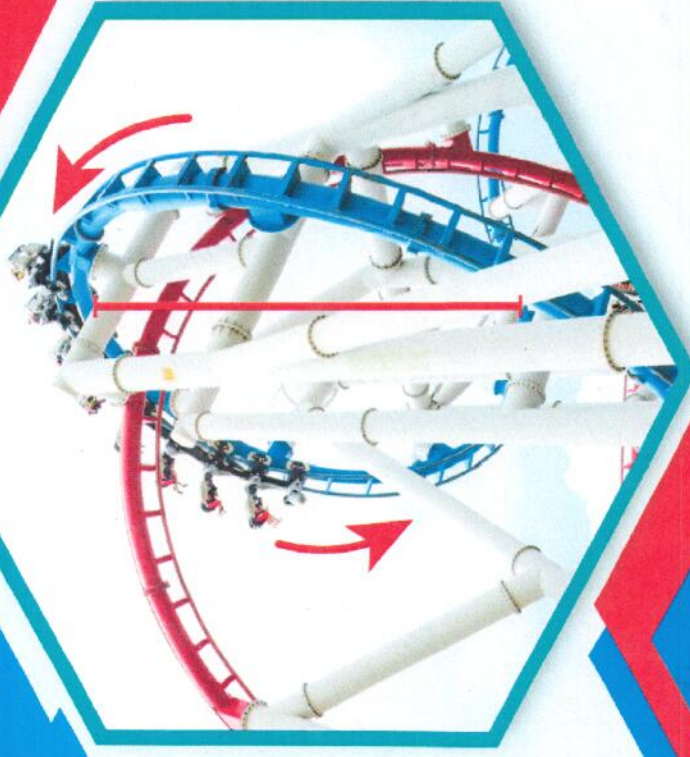
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# Cálculo diferencial







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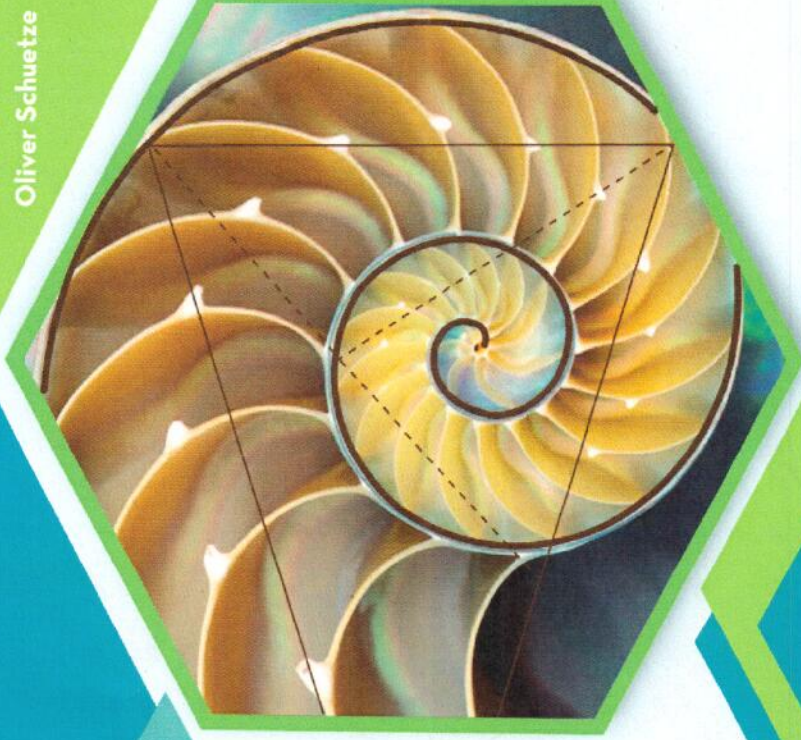
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# Cálculo integral





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Review

# The Averaged Hausdorff Distances in Multi-Objective Optimization: A Review

Johan M. Bogoya <sup>1</sup>, Andrés Vargas <sup>1</sup> and Oliver Schütze <sup>2,3,\*</sup>

<sup>1</sup> Departamento de Matemáticas, Pontificia Universidad Javeriana, Cra. 7 N. 40-62, Bogotá D.C. 111321, Colombia; jbogoya@javeriana.edu.co (J.M.B.); a.vargas@javeriana.edu.co (A.V.)

<sup>2</sup> Computer Science Department, CINVESTAV-IPN, Av. IPN 2508, Col. San Pedro Zacatenco, Mexico City 07360, Mexico

<sup>3</sup> Dr. Rodolfo Quintero Ramirez Chair, UAM Cuajimalpa, Mexico City 05348, Mexico

\* Correspondence: schuetze@cs.cinvestav.mx

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**Abstract:** A brief but comprehensive review of the averaged Hausdorff distances that have recently been introduced as quality indicators in multi-objective optimization problems (MOPs) is presented. First, we introduce all the necessary preliminaries, definitions, and known properties of these distances in order to provide a state-of-the-art overview of their behavior from a theoretical point of view. The presentation treats separately the definitions of the  $(p, q)$ -distances  $GD_{p,q}$ ,  $IGD_{p,q}$ , and  $\Delta_{p,q}$  for finite sets and their generalization for arbitrary measurable sets that covers as an important example the case of continuous sets. Among the presented results, we highlight the rigorous consideration of metric properties of these definitions, including a proof of the triangle inequality for distances between disjoint subsets when  $p, q \geq 1$ , and the study of the behavior of associated indicators with respect to the notion of compliance to Pareto optimality. Illustration of these results in particular situations are also provided. Finally, we discuss a collection of examples and numerical results obtained for the discrete and continuous incarnations of these distances that allow for an evaluation of their usefulness in concrete situations and for some interesting conclusions at the end, justifying their use and further study.

**Keywords:** Averaged Hausdorff distance; evolutionary multi-objective optimization; Pareto compliance; performance indicator; power means

## 1. Introduction

In many real-world applications, the problem of concurrent or simultaneous optimization of several objectives is an essential task known as a multi-objective optimization problem (MOP). One important problem in multi-objective optimization is to compute a suitable finite size approximation of the solution set of a given MOP, the so-called Pareto set and its image, the Pareto front.

The Hausdorff distance  $d_H$  (e.g., Reference [1]) measures how far two subsets of a metric space are from each other. Due to its properties, it is frequently used in many research areas such as computer vision [2–4], fractal geometry [5], the numerical computation of attractors in dynamical systems [6–8], or convergence of multi-objective algorithms to the Pareto set/front of a given multi-objective optimization problem [9–15]. One possible drawback of the classical Hausdorff distance, however, is that it punishes single outliers which leads to inequitable performance evaluations in some cases. As one example, we mention here multi-objective evolutionary algorithms. On the one hand, such algorithms are known to be very effective in the (global) approximation of the Pareto set/front. On the other hand, it is also known that the final approximations (populations) may contain some outliers (e.g., Reference [16]). For such cases, the Hausdorff distance may indicate a “bad” match of population



# The Assessed Handbook of the Multi-Objective Optimization Algorithm

John G. Thompson, Department of Mathematics, University of Illinois at Chicago

1. Introduction
2. The Assessed Handbook of the Multi-Objective Optimization Algorithm
3. The Assessed Handbook of the Multi-Objective Optimization Algorithm
4. The Assessed Handbook of the Multi-Objective Optimization Algorithm
5. The Assessed Handbook of the Multi-Objective Optimization Algorithm

Abstract: This handbook provides a comprehensive overview of the Multi-Objective Optimization Algorithm, including its theoretical foundations, practical applications, and implementation details.

The Multi-Objective Optimization Algorithm is a powerful tool for solving complex optimization problems involving multiple conflicting objectives. This handbook provides a detailed overview of the algorithm, including its theoretical foundations, practical applications, and implementation details. The algorithm is designed to find the Pareto frontier of a multi-objective optimization problem, which is the set of all non-dominated solutions. The handbook is organized into five main sections: Introduction, The Assessed Handbook of the Multi-Objective Optimization Algorithm, The Assessed Handbook of the Multi-Objective Optimization Algorithm, The Assessed Handbook of the Multi-Objective Optimization Algorithm, and The Assessed Handbook of the Multi-Objective Optimization Algorithm. Each section provides a comprehensive overview of the algorithm, including its theoretical foundations, practical applications, and implementation details. The handbook is intended for researchers, practitioners, and students in the field of multi-objective optimization.

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## 1. Introduction

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Article

# Variation Rate to Maintain Diversity in Decision Space within Multi-Objective Evolutionary Algorithms<sup>†</sup>

Oliver Cuate<sup>1,\*</sup> and Oliver Schütze<sup>2,\*</sup>

<sup>1</sup> Computer Science Department, Cinvestav-IPN, 07360 Mexico City, Mexico

<sup>2</sup> Dr. Rodolfo Quintero Ramirez Chair, UAM Cuajimalpa, 05370 Mexico City, Mexico

\* Correspondence: ocuate@computacion.cs.cinvestav.mx (O.C.); schuetze@cs.cinvestav.mx (O.S.)

<sup>†</sup> This paper is an extended version of our paper published in the 10th International Conference on Evolutionary Multi-Criterion Optimization (EMO 2019), East Lansing, MI, USA, in 10–13 March 2019.

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**Abstract:** The performance of a multi-objective evolutionary algorithm (MOEA) is in most cases measured in terms of the populations' approximation quality in objective space. As a consequence, most MOEAs focus on such approximations while neglecting the distribution of the individuals of their populations in decision space. This, however, represents a potential shortcoming in certain applications as in many cases one can obtain the same or very similar qualities (measured in objective space) in several ways (measured in decision space). Hence, a high diversity in decision space may represent valuable information for the decision maker for the realization of a given project. In this paper, we propose the Variation Rate, a heuristic selection strategy that aims to maintain diversity both in decision and objective space. The core of this strategy is the proper combination of the averaged distance applied in variable space together with the diversity mechanism in objective space that is used within a chosen MOEA. To show the applicability of the method, we propose the resulting selection strategies for some of the most representative state-of-the-art MOEAs and show numerical results on several benchmark problems. The results demonstrate that the consideration of the Variation Rate can greatly enhance the diversity in decision space for all considered algorithms and problems without a significant loss in the approximation qualities in objective space.

**Keywords:** evolutionary computation; multi-objective optimization; decision space diversity

## 1. Introduction

In many areas such as Economy, Finance, or Industry, the problem arises naturally that several conflicting objectives have to be optimized concurrently [1,2]. Such problems are called multi-objective optimization problems (MOPs) in literature [3–5]. The solution set of an MOP (in decision space) is called the Pareto set and its image (defined in objective space) the Pareto front. One important characteristic of continuous MOPs is that their Pareto sets and fronts typically form  $(k - 1)$ -dimensional objects, where  $k$  is the number of objectives considered in the problem. In many applications, the Pareto front is of primary interest as it contains information about the desired qualities of each selected solution. As a consequence, almost all existing MOEAs focus on approximations in objective space while entirely neglecting the distribution of the individuals in decision space. There exist, however, also applications where the values of the solutions in decision variable space are of great importance. As an example, consider that the amount of a certain resource (i.e., the value of a variable  $x_i$ ) used to obtain the desired quality (measured in objective space) is important. For two solutions that are equal or similar in objective space, one may prefer the one that has a lower value of  $x_i$ . Another example is that the variable  $x_i$  could represent the launch date of a project, as, e.g., in [6] in the context of space mission design. For such problems, different values of  $x_i$  directly relate to different timescales in the

# Variation in the Maintenance of Health Insurance: A Review

John A. DiMatteo, MD, PhD

From the Department of Health, Behavior, and Society, Johns Hopkins University, Baltimore, Md. (Dr. DiMatteo).  
Address correspondence and reprint requests to Dr. DiMatteo at the Department of Health, Behavior, and Society, Johns Hopkins University, 615 North Wolfe Street, Baltimore, Md. 21205.  
Received for publication May 15, 1997; accepted for publication July 15, 1997.

**Abstract:** The purpose of this review is to provide a comprehensive overview of the literature on the maintenance of health insurance.

After a brief review of the importance of health insurance, the review focuses on the maintenance of health insurance. The review is organized into three main sections: (1) the importance of health insurance, (2) the maintenance of health insurance, and (3) the impact of health insurance on health status. The review begins by discussing the importance of health insurance in the United States. It then discusses the maintenance of health insurance, focusing on the factors that influence the decision to maintain health insurance. Finally, the review discusses the impact of health insurance on health status, focusing on the impact of health insurance on the management of chronic diseases.

**Key Words:** Health insurance, maintenance, chronic diseases, health status.

## Introduction

In the United States, health insurance is a critical component of the health care system. It provides the financial means by which individuals can access the health care services they need. Without health insurance, many individuals would be unable to afford the care they need, and their health status would likely be compromised. The purpose of this review is to provide a comprehensive overview of the literature on the maintenance of health insurance. The review is organized into three main sections: (1) the importance of health insurance, (2) the maintenance of health insurance, and (3) the impact of health insurance on health status. The review begins by discussing the importance of health insurance in the United States. It then discusses the maintenance of health insurance, focusing on the factors that influence the decision to maintain health insurance. Finally, the review discusses the impact of health insurance on health status, focusing on the impact of health insurance on the management of chronic diseases.





# A neural network-evolutionary computational framework for remaining useful life estimation of mechanical systems

David Laredo<sup>a</sup>, Zhaoyin Chen<sup>a</sup>, Oliver Schütze<sup>b,c</sup>, Jian-Qiao Sun<sup>a,\*</sup>

<sup>a</sup> Department of Mechanical Engineering, School of Engineering, University of California, Merced, CA 95343, USA

<sup>b</sup> Department of Computer Science, CINVESTAV, Mexico City, Mexico

<sup>c</sup> Rodolfo Quintero Chair, UAM Cuajimalpa, Mexico

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## ABSTRACT

This paper presents a framework for estimating the remaining useful life (RUL) of mechanical systems. The framework consists of a multi-layer perceptron and an evolutionary algorithm for optimizing the data-related parameters. The framework makes use of a strided time window along with a piecewise linear model to estimate the RUL for each mechanical component. Tuning the data-related parameters in the optimization framework allows for the use of simple models, e.g. neural networks with few hidden layers and few neurons at each layer, which may be deployed in environments with limited resources such as embedded systems. The proposed method is evaluated on the publicly available C-MAPSS dataset. The accuracy of the proposed method is compared against other state-of-the-art methods in the literature. The proposed method is shown to perform better than the compared methods while making use of a compact model.

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## 1. Introduction

Traditionally, maintenance of mechanical systems has been carried out based on scheduling strategies. Such strategies are often costly and less capable of meeting the increasing demand of efficiency and reliability (Gebrael, Lawley, Liu, & Ryan, 2005; Zaidan, Mills, & Harrison, 2013). Condition based maintenance (CBM) also known as intelligent prognostics and health management (PHM) allows for maintenance based on the current health of the system, thus cutting down the costs and increasing the reliability of the system (Zhao, Bin, Wang, & Lu, 2017). Here, we refer to prognostics as the estimation of remaining useful life of a system. The remaining useful life (RUL) of the system can be estimated based on the historical data. This data-driven approach can help optimize maintenance schedules to avoid engineering failures and to save costs (Lee et al., 2014).

The existing PHM methods can be grouped into three different categories: model-based (Yu & Kuffi, 2001), data-driven (Liu & Wang, 2009; Mosallam, Medjaher, & Zerhouni, 2013) and hybrid approaches (Liu, Wang, & Yang, 2012; Pecht & Jaai, 2010). Model-based approaches attempt to incorporate physical models of the system into the estimation of the RUL. If the system degradation is modeled precisely, model-based approaches usually exhibit better performance than data-driven approaches

(Qian, Yan, & Gao, 2017). This comes at the expense of having extensive a priori knowledge of the underlying system and having a fine-grained model of the system, which can involve expensive computations. On the other hand, data-driven approaches use pattern recognition to detect changes in system states. Data-driven approaches are appropriate when the understanding of the first principles of the system dynamics is not comprehensive or when the system is sufficiently complex such as jet engines, car engines and complex machineries, for which it is prohibitively difficult to develop an accurate model.

Common disadvantages for the data-driven approaches are that they usually exhibit wider confidence intervals than model-based approaches and that a fair amount of data is required for training. Many data-driven algorithms have been proposed. Good prognostics results have been achieved. Among the most popular algorithms we can find artificial neural networks (ANNs) (Gebrael, Lawley, Li, & Parmeshwaran, 2004), support vector machine (SVM) (Benkedjouh, Medjaher, Zerhouni, & Rechak, 2013), Markov hidden chains (MHC) (Dong & He, 2007) and so on. Over the past few years, data-driven approaches have gained more attention in the PHM community. A number of machine learning techniques, especially neural networks, have been applied successfully to estimate the RUL of diverse mechanical systems. ANNs have demonstrated good performance in modeling highly nonlinear, complex, multi-dimensional systems without any prior knowledge on the system behavior (Li, Ding, & Sun, 2018). While

\* Corresponding author.

E-mail address: [jqsun@ucmerced.edu](mailto:jqsun@ucmerced.edu) (J.-Q. Sun).







# Pareto Explorer: a global/local exploration tool for many-objective optimization problems

Oliver Schütze<sup>a,b</sup>, Oliver Cuate<sup>a</sup>, Adanay Martín<sup>a</sup>, Sebastian Peitz<sup>c</sup> and Michael Dellnitz<sup>c</sup>

<sup>a</sup>Computer Science Department, Cinvestav-IPN, Mexico City, Mexico; <sup>b</sup>UAM Cuajimalpa, Mexico City, Mexico;

<sup>c</sup>Institut für Industriemathematik, Paderborn University, Paderborn, Germany

## ABSTRACT

Multi-objective optimization is an active field of research that has many applications. Owing to its success and because decision-making processes are becoming more and more complex, there is a recent trend for incorporating many objectives into such problems. The challenge with such problems, however, is that the dimensions of the solution sets—the so-called Pareto sets and fronts—grow with the number of objectives. It is thus no longer possible to compute or to approximate the entire solution set of a given problem that contains many (e.g. more than three) objectives. On the other hand, the computation of single solutions (e.g. via scalarization methods) leads to unsatisfying results in many cases, even if user preferences are incorporated. In this article, the Pareto Explorer tool is presented—a global/local exploration tool for the treatment of many-objective optimization problems (MaOPs). In the first step, a solution of the problem is computed via a global search algorithm that ideally already includes user preferences. In the second step, a local search along the Pareto set/front of the given MaOP is performed in user specified directions. For this, several continuation-like procedures are proposed that can incorporate preferences defined in decision, objective, or in weight space. The applicability and usefulness of Pareto Explorer is demonstrated on benchmark problems as well as on an application from industrial laundry design.

## ARTICLE HISTORY

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## KEYWORDS

Multi-objective optimization; many-objective optimization; decision making; predictor–corrector method

## 1. Introduction

In many applications, the problem arises that several objectives have to be optimized concurrently, leading to *multi-objective optimization problems* (MOPs). Owing to the increasing complexity of practical problems, decision-making processes are becoming more and more sophisticated. Motivated by advances in the design of algorithms for the numerical treatment of MOPs with few objectives and their huge success in applications, there is a recent trend for including more objectives in the optimization process. As an example, in the design of a car there are factors such as cost, energy consumption, emission rates, maximal speed and acceleration, as well as many other indicators related to performance, style and sustainability, that are certainly of high interest in the decision-making process. Thus, all of them are possible objectives for the related decision making. There is, however, one important characteristic of MOPs that has to be considered in their treatment, namely that the solution set (the Pareto set or its image, the Pareto front) typically form  $(k - 1)$ -dimensional objects, where  $k$  is the number of objectives. This fact has a crucial impact on the approximation qualities

**CONTACT** Oliver Schütze  [schuetze@cs.cinvestav.mx](mailto:schuetze@cs.cinvestav.mx).

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# Multi-objective and many objective design of plastic injection molding process

Alejandro Alvarado-Iniesta<sup>1</sup> · Oliver Cuate<sup>2</sup> · Oliver Schütze<sup>3</sup>

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## Abstract

Plastic injection molding is one of the most used manufacturing processes capable of producing flexible and economical parts at a large scale. Since this is a highly complex process, it is a natural consequence that there are many conflicting objectives that are worth considering in the design of such process. Problems where more than three objectives are being considered at the same time are termed many objective problems (MaOPs) in literature. Unlike for multi-objective problems (MOPs, problems with two or three objectives), there is no consensus of how to find ideal solutions for general MaOPs. In this paper, the multi-objective and many objective design of a plastic injection molding process is addressed. To accomplish this task, the two main contributions of this work are as follows: first, a new optimization model that contains up to seven objectives is proposed. That is, for the first time, it is considered the many objective design of a plastic injection process. Second, the usefulness of the Pareto Explorer, a global/local exploration tool for MaOPs, in the current context is demonstrated. For this, the complete seven-objective optimization problem on several selected scenarios related to the hypothetical decision making of a plastic gear is considered.

**Keywords** Plastic injection molding · Multi-objective optimization · Many objective optimization · Decision making

## 1 Introduction

Plastic injection molding (PIM) is one of the major processes in polymer processing capable of producing parts with complex shapes at a relatively low cost. Many daily use products, e.g., electronics devices, appliances, and packaging, rely on the technology and production of the

PIM industry. PIM is a process that consists of six phases: clamping, filling, packing, cooling, opening, and ejecting. All phases are not independent with each other; an improper clamping setting may result in a failing filling phase, and so on, affecting the performance of the overall process. Usually, the performance of a PIM process is measured by outcomes such as qualities of the parts, i.e., appearance characteristics, and functional properties [27, 28, 32, 52, 53, 62], and productivity indicators, e.g., production cost, cycle time, and energy consumption [27, 33, 61]. Undoubtedly, there is a trade-off between quality and productivity (e.g., a high-quality product rarely results in a low production cost and vice versa); therefore, the PIM process can be viewed as a problem where several incommensurate and competing outcomes need to be simultaneously satisfied.

Such problems are termed *multi-objective optimization problems* (MOPs) in literature if two or three objectives are being considered, and *many objective optimization problems* (MaOPs) if more than three objectives are under consideration. Although MOPs and MaOPs are in principle defined equally— $k$  objectives have to be optimized concurrently—these two distinct terms are used since the solution sets of M(a)OPs, the so-called Pareto sets, respectively, their images, the Pareto fronts, typically form

✉ Alejandro Alvarado-Iniesta  
alejandro.alvarado@uacj.mx

Oliver Cuate  
ocuate@computacion.cs.cinvestav.mx

Oliver Schütze  
schuetze@cs.cinvestav.mx

<sup>1</sup> Department of Industrial and Manufacturing Engineering, Universidad Autónoma de Ciudad Juárez, Ciudad Juárez, Chihuahua, Mexico

<sup>2</sup> Computer Science Department, Cinvestav-IPN, Mexico City, Mexico

<sup>3</sup> Department of Applied Mathematics and Systems, Dr. Rodolfo Quintero Chair, UAM Cuajimalpa, Mexico City, Mexico





# The Set-Based Hypervolume Newton Method for Bi-Objective Optimization

Víctor Adrián Sosa Hernández<sup>✉</sup>, Oliver Schütze, Hao Wang, André Deutz, and Michael Emmerich

**Abstract**—In this paper, we propagate the use of a set-based Newton method that enables computing a finite size approximation of the Pareto front (PF) of a given twice continuously differentiable bi-objective optimization problem (BOP). To this end, we first derive analytically the Hessian matrix of the hypervolume indicator, a widely used performance indicator for PF approximation sets. Based on this, we propose the hypervolume Newton method (HNM) for hypervolume maximization of a given set of candidate solutions. We first address unconstrained BOPs and focus further on first attempts for the treatment of inequality constrained problems. The resulting method may even converge quadratically to the optimal solution, however, this property is—as for all Newton methods—of local nature. We hence propose as a next step a hybrid of HNM and an evolutionary strategy in order to obtain a fast and reliable algorithm for the treatment of such problems. The strengths of both HNM and hybrid are tested on several benchmark problems and comparisons of the hybrid to state-of-the-art evolutionary algorithms for hypervolume maximization are presented.

**Index Terms**—Hessian matrix, hypervolume indicator, memetic algorithms, Newton method, set-based local search.

## I. INTRODUCTION

IN MANY applications, the problem arises where several objectives have to be optimized concurrently leading to a multiobjective optimization problem (MOP). One important characteristic of MOPs is that their solutions sets, the Pareto sets (PSs), typically (under certain conditions when the MOP is continuous and nondegenerate) form objects of dimension  $(k - 1)$ , where  $k$  is the number of objectives involved in the problem. In many cases, the decision maker is interested in a

suitable finite size approximation of the entire PS, namely, its image, and the Pareto front (PF) [1]. To address this problem, specialized evolutionary algorithms are predominantly used [2]–[8]. Such methods are capable of returning a finite size approximation of the entire set of interest in one single run of the algorithm, and as they are very robust and of global nature [9], [10]. One major drawback of all such methods, however, is that they are quite costly in terms of function evaluations. That is, they typically require quite a few function evaluations in order to obtain suitable approximations. Moreover, they do not converge toward the optimal solution in the mathematical sense, instead, their populations may get stuck in nonoptimal (but in most cases in nearly optimal) regions.

The inclusion of gradient information has long been considered as a remedy to imprecision [11], [12]. However, the initially proposed methods were only able to improve the closeness to the PF, but not the diversity of approximations. To also include the diversity in gradient-based search for PFs, directed search methods were proposed in [13] and [14], which allow not only to move toward the PF based on local gradient information but also along this set in order to increase diversity. Alternatively, it proposed the use of the hypervolume gradient defined on an entire population to find diverse set approximations to the PF (see [15], [16]).

In this paper, we first present the hypervolume Newton method (HNM) for hypervolume maximization. The hypervolume [17] is a widely used indicator to measure the approximation quality of a set. The method allows for a finite size approximation of the PF of a bi-objective optimization problem (BOP) in one run of the algorithm. To this end, we first derive an analytical expression of the Hessian matrix of the hypervolume indicator (which is being considered as a particular scalar optimization problem defined on sets). Based on this, we formulate the set-based HNM first for unconstrained and later on for inequality constrained BOP. The resulting method is defined on entire sets and yields up to quadratic convergence rates. The method, however, is as all Newton methods of local nature and requires good starting sets. That is why in the second step of this paper, we propose a hybrid evolutionary algorithm that utilizes HNM as a local search engine. Numerical results against state-of-the-art evolutionary algorithms for hypervolume indicator maximization demonstrate the benefit of the novel approach.

A first study of this paper can be found in [18], where an HNM is proposed for unconstrained BOPs, albeit using finite difference approximations of the Hessians. The idea to use the

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V. A. Sosa Hernández is with the School of Engineering and Sciences, Tecnológico de Monterrey, Atizapán de Zaragoza 52926, Mexico (e-mail: vsosa@tec.mx).

O. Schütze is with the Department of Computer Science, CINVESTAV-IPN, Mexico City 07360, Mexico, and also with the Department of Applied Mathematics and Systems, UAM Cuajimalpa, Mexico City 05370, Mexico (e-mail: schuetze@cs.cinvestav.mx).

H. Wang, A. Deutz, and M. Emmerich are with the Leiden Institute of Advanced Computer Science, Leiden University, 2333 CA Leiden, The Netherlands (e-mail: h.wang@liacs.leidenuniv.nl; a.h.deutz@liacs.leidenuniv.nl; m.t.m.emmerich@liacs.leidenuniv.nl).

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# Pareto Explorer for Solving Real World Applications

Oliver Cuate<sup>1</sup> and Oliver Schtze<sup>1,2</sup>

<sup>1</sup> CINESTAV, Computer Science Department, Cinvestav-IPN, 07360 Mexico City, Mexico

ocuate@computacion.cs.cinvestav.mx

<sup>2</sup> Dr. Rodolfo Quintero Ramirez Chair, UAM Cuajimalpa, 05370 Mexico City, Mexico schuetze@cs.cinvestav.mx

**Abstract.** An important characteristic of the Multi-objective Optimization Problems (MOPs) is that their solution sets typically form a  $(k-1)$ -dimensional object where  $k$  is the number of objectives involved in the MOP. Thus, it is only possible to approximate the entire set of interest for a relatively few numbers of objectives (say,  $k = 3$  or  $4$ ). In this work, we address the numerical treatment of MOPs with more than four objectives which are termed as Many Objective Optimization Problems (MaOPs). Such problems have recently caught the interest in the industry as the decision-making processes are getting more and more complex. The recently proposed Pareto Explorer (PE) method raises as a solution for the MaOPs, it is conceived as a global/local exploration tool which consists of two principal phases: obtaining a global optimal solution for a given MaOP, and the local exploration of optimal solutions based on the preferences of a decision-maker. In this work, we demonstrate the effectiveness of PE for solving real-world applications.

**Keywords:** Many objective optimization · Interactive method · Decision-making · Continuation method.

## 1 Introduction

In many applications, several objectives have to be optimized concurrently leading to a *multi-objective optimization problem* (MOP). Due to the increasing complexity of practical problems, decision-making processes are getting more and more sophisticated. Motivated by the advances in the design of algorithms for the numerical treatment of MOPs with few objectives and their huge success in applications, there is a recent trend to include more objectives into the optimization process. Due to this reason, MOPs with more than four objectives are often termed *many objective problems* (MaOPs) in the literature as they require a different numerical treatment than problems with two to four objectives.

However, there exist real-world problems where the decision-maker has some knowledge about the problem or she/he wants to obtain optimal solutions with specific characteristics instead of a vast set of alternatives. Reference point methods are useful for this scenario, where the idea is to get the closest solution to

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# Variation Rate: An Alternative to Maintain Diversity in Decision Space for Multi-objective Evolutionary Algorithms

Oliver Cuate<sup>1</sup>(✉) and Oliver Schütze<sup>2</sup>

<sup>1</sup> Computer Science Department, Cinvestav-IPN, Av. Instituto Politécnico Nacional No. 2508 Col. San Pedro Zacatenco, Mexico City, Mexico

ocuate@computacion.cs.cinvestav.mx

<sup>2</sup> Department of Applied Mathematics and Systems,

UAM Cuajimalpa Dr. Rodolfo Quintero Chair., Mexico City, Mexico

schuetze@cs.cinvestav.mx

**Abstract.** In almost all cases the performance of a multi-objective evolutionary algorithm (MOEA) is measured in terms of its approximation quality in objective space. As a consequence, most MOEAs focus on such approximations while neglecting the distribution of the individuals in decision space. This, however, represents a potential shortcoming in certain applications as in many cases one can obtain the same or a very similar qualities (measured in objective space) in several ways (measured in decision space) which may be very valuable information for the decision maker for the realization of a project.

In this work, we propose the variable-NSGA-III (vNSGA-III) an algorithm that performs an exploration both in objective and decision space. The idea behind this algorithm is the so-called variation rate, a heuristic that can easily be integrated into other MOEAs as it is free of additional design parameters. We demonstrate the effectiveness of our approach on several benchmark problems, where we show that, compared to other methods, we significantly improve the approximation quality in decision space without any loss in the quality in objective space.

**Keywords:** Evolutionary computation · Multi-objective optimization · Decision space diversity

## 1 Introduction

In many areas such as Economy, Finance, or Industry the problem arises naturally that several conflicting objectives have to be optimized concurrently. This leads to multi-objective optimization problems (MOPs). The solution of this kind of problems is a set of vectors that are incomparable to each other in terms

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of their objective values. For some of these problems obtaining the greatest benefit from limited resources is essential. Such resources are typically represented as the variables of the problem, as the objective functions depends on them. Although by using constraints it is possible to control the value of decision variables, this would entail the loss of optimal solutions and that is not desirable for the decision-making process. For instance, in real-world problems where the value of some variables is crucial, the decision maker may prefer, among the set of optimal solutions, those that are easiest to implement as this can mean a saving in resources.

However, in almost all cases the performance of a MOEA is only measured in terms of its approximation quality in objective space. As a consequence, most MOEAs focus on such approximations while neglecting the distribution of the individuals in decision space. This represents a potential shortcoming in certain applications as in many cases one can obtain the same or a very similar quality (measured in objective space) in several ways (measured in decision space) which may be very valuable information for the decision maker for the realization of a project. In this context, there exists an additional challenge in solving a MOP, since we must find an approximation to the optimal set both in objective and decision space, in order to provide all these possible regions to the decision maker.

In this work, we propose the variable-NSGA-III (vNSGA-III) an algorithm that performs an exploration both in objective and decision space. The idea behind this algorithm is the so-called variation rate, a heuristic that can easily be integrated into other MOEAs as it is free of additional design parameters. We demonstrate the effectiveness of our approach on several benchmark problems, where we show that, compared to other methods, we significantly improve the approximation quality in decision space without any loss in the quality in objective space.

The rest of the paper is organized as follows, in Sect. 2, we present the background and the related work. In Sect. 3, a detailed description of the proposed algorithm (along with pseudo-codes) is presented. In Sect. 4, numerical results are provided. Finally, in Sect. 5, we discuss the advantages of the proposed algorithm and we discuss the possible future improvements to the algorithm.

## 2 Background and Related Work

Optimization refers to finding the best possible solution to a problem given a set of constraints [2]. MOP refers to the simultaneous optimization of multiple and usually conflicting objectives; as a result, a set of optimal solutions are obtained instead of having a single optimal solution. The MOP with  $k$  objectives is mathematical defined as:

$$\min_{x \in D} F(x), \quad (1)$$

where  $D \subset \mathbb{R}^n$  is the domain and  $F : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^k$  is the objective function.

The optimality of a MOP is defined by the concept of *dominance*. Let  $v, w \in \mathbb{R}^k$ , the vector  $v$  is *less than*  $w$  ( $v <_p w$ ), if  $v_i < w_i$  for all  $i \in \{1, \dots, k\}$ ; the



# Toward a New Family of Hybrid Evolutionary Algorithms

Lourdes Uribe<sup>1,2(✉)</sup>, Oliver Schütze<sup>1,2</sup>, and Adriana Lara<sup>1,2</sup>

<sup>1</sup> ESFM, Instituto Politécnico Nacional,  
Av. Instituto Politécnico Nacional Edif. 9, Unidad Profesional Adolfo López Mateos,  
Zacatenco, Mexico

{lourdesur,adriana}@esfm.ipn.mx, schuetze@cs.cinvestav.mx

<sup>2</sup> Department of Applied Mathematics and Systems,  
UAM Cuajimalpa, Dr. Rodolfo Quintero Chair, Mexico City, Mexico

**Abstract.** Multi-objective optimization problems (MOPs) arise in a natural way in diverse knowledge areas. Multi-objective evolutionary algorithms (MOEAs) have been applied successfully to solve this type of optimization problems over the last two decades. However, until now MOEAs need quite a few resources in order to obtain acceptable Pareto set/front approximations. Even more, in certain cases when the search space is highly constrained, MOEAs may have troubles when approximating the solution set. When dealing with constrained MOPs (CMOPs), MOEAs usually apply penalization methods. One possibility to overcome these situations is the hybridization of MOEAs with local search operators. If the local search operator is based on classical mathematical programming, gradient information is used, leading to a relatively high computational cost. In this work, we give an overview of our recently proposed constraint handling methods and their corresponding hybrid algorithms. These methods have specific mechanisms that deal with the constraints in a wiser way without increasing their cost. Both methods do not explicitly compute the gradients but extract this information in the best manner out of the current population of the MOEAs. We conjecture that these techniques will allow for the fast and reliable treatment of CMOPs in the near future. Numerical results indicate that these ideas already yield competitive results in many cases.

**Keywords:** Multi-objective optimization ·  
Evolutionary computation · Mathematical programming ·  
Hybrid meta-heuristics

## 1 Introduction

In many engineering applications one is faced with the problem that several objectives have to be optimized concurrently resulting in a multi-objective optimization problem (MOP). For the treatment of MOPs, traditional optimization

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techniques establish effective search directions by using differentiability properties of the objective functions. These directions should be able to (at least locally) lead toward better solutions with respect to the objective value. The computation of these proper multi-objective search directions requires, in general, the numerical approximation of the derivatives. These search directions produce a sequence of trial points which eventually converge to one single local optimum point of the problem. It is worth to notice that the solution of a MOP, the Pareto set (PS), typically forms a  $(k - 1)$ -dimensional object where  $k$  is the number of objectives.

Multi-objective evolutionary algorithms (MOEAs) have caught the interest of many researchers (see, e.g. [4,6,30]) over the last two decades. Some reasons for this include that MOEAs are of global nature, and hence, they do not depend on the initial population. Further, due to their set based approach they compute a finite size approximation of the entire PS in one single run of the algorithm. Also, MOEAs do not require gradient information. Recently, hybrid algorithms have gained popularity. They combine gradient-based local search with MOEAs. In particular, for unconstrained MOPs we refer to [13,20,23,24], and to [18, 25] for constrained MOPs. Also, hybrid MOEAs with non-gradient based local search can be found (e.g., [28] and [29]). Designing these hybrid algorithms is not a direct process since MOEAs are stochastic by nature, and making solution improvements in a deterministic way can affect the convergence of the set based algorithm. Besides, the cost of performing gradient-based local search could be excessive, considering the particular improvement. For this reason, designing effective local search procedures is highly relevant. For the case of constrained MOPs, hybrid algorithms mostly rely on their evolutionary part to manage the feasibility of the solutions, and do not involve constraint function information during the evolutionary process.

In [20], an analysis about the behavior of multi-objective stochastic local search (MOSLS) was presented. This analysis showed that a pressure both toward and along the Pareto front (PF) is already inherent for unconstrained problems. **For the constrained case however, this behavior is not preserved.** Although, based on the Karush-Kuhn Tucker (KKT) equations for optimality, one can identify subspaces that allow a movement along the Pareto front for points that are near to the solution set. Since gradients are required to obtain these subspaces, an increment in the computational cost is expected. In this work, we present an overview of our recently proposed constraint handling methods and their corresponding hybrid algorithms. These methods have specific mechanisms, that take advantage of these subspaces, i.e., they move along the PF. In addition, these methods extract information in a best manner out of the current population of the MOEAs. In this way, we are able to move along the Pareto front and maintain a low computational cost.

We first present the Subspace Mutation Operator (SPM) as an alternative variation operator for MOEAs. Classically, mutation has been guided for “small-moves heuristics” [22] or differences on the objective values [15]. For the constrained case, the constraints management is left to the selection process in the





# A New Hybrid Metaheuristic for Equality Constrained Bi-objective Optimization Problems

Oliver Cuate<sup>1</sup>(✉), Lourdes Uribe<sup>2</sup>, Antonin Ponsich<sup>3</sup>, Adriana Lara<sup>2</sup>,  
Fernanda Beltran<sup>1</sup>, Alberto Rodríguez Sánchez<sup>3</sup>, and Oliver Schütze<sup>4</sup>

<sup>1</sup> Computer Science Department, Cinvestav-IPN,

Av. Instituto Politécnico Nacional No. 2508, Col. San Pedro Zacatenco, Mexico

{ocuate,mbeltran}@computacion.cs.cinvestav.mx

<sup>2</sup> ESFM, Instituto Politécnico Nacional,

Av. Instituto Politécnico Nacional Edif. 9, Unidad Profesional Adolfo López Mateos,  
Zacatenco, Mexico

{lourdesur,adriana}@esfm.ipn.mx

<sup>3</sup> Universidad Autónoma Metropolitana Azcapotzalco,

Av. San Pablo No. 180, Col. Reynosa Tamaulipas, Azcapotzalco, Mexico

{aspo,al2161801914}@azc.uam.mx

<sup>4</sup> Department of Applied Mathematics and Systems,

UAM Cuajimalpa, Dr. Rodolfo Quintero Chair, Mexico City, Mexico

schuetze@cs.cinvestav.mx

**Abstract.** The recently proposed Pareto Tracer method is an effective numerical continuation technique which allows performing movements along the set of KKT points of a given multi-objective optimization problem. The nature of this predictor-corrector method leads to constructing solutions along the Pareto set/front numerically: it applies to higher dimensions and can handle box and equality constraints. We argue that the right hybridization of multi-objective evolutionary algorithms together with specific continuation methods leads to fast and reliable algorithms. Moreover, due to the continuation technique, the resulting hybrid algorithm could have a certain advantage when handling, in particular, equality constraints. In this paper, we make the first effort to hybridize NSGA-II with the Pareto Tracer. To support our claims, we present some numerical results on continuously differentiable equality constrained bi-objective optimization test problems, to show that the resulting hybrid NSGAII/PT is highly competitive against some state-of-the-art algorithms for constrained optimization. Finally, we stress that the chosen approach could be applied to a more significant number of objectives with some adaptations of the algorithm, leading to a very promising research topic.

**Keywords:** Multi-objective optimization · Evolutionary algorithms · Continuation methods · Hybrid algorithms

The authors acknowledge support for CONACyT project No. 285599 and IPN project SIP20181450.



## 1 Introduction

In many different applications, the problem arises that several conflicting objectives have to be optimized concurrently [6,19]. Such problems are termed as multi-objective optimization problems (MOPs) in literature. One important characteristic of a MOP is that its solution set, the *Pareto set*, and its image, the *Pareto front*, typically form locally  $(k - 1)$ -dimensional manifolds, where  $k$  is the number of objectives of the MOP. Since these sets can not be computed analytically—apart from simple academic problems—the question arises how to compute suitable finite size approximations of them.

So far, many numerical methods for the treatment of a given MOP have been proposed. There exist, for instance, specialized evolutionary algorithms (EAs), called multi-objective evolutionary algorithms (MOEAs), that have caught the interest of many researchers in the recent past (e.g. [1,4,7,10,11,26]). Reasons for this include the global approach of these population-based methods, their relatively low assumptions on the model, their high robustness, and that they are capable of delivering a finite size approximation of the entire set of interest in one single run of the algorithm. The latter is because the whole population will evolve. These significant advantages, however, come at a certain price. It is widely accepted that MOEAs are very good in detecting promising regions and are highly efficient in computing rough approximations of the solution sets. However, they typically need quite a few function evaluations to obtain reasonable estimates of the Pareto set/front. This drawback gets even more significant in case the model contains complex constraints and/or the decision variable space is high-dimensional. As a possible remedy, researchers have hybridized evolutionary strategies with local search techniques mainly coming from mathematical programming leading to so-called memetic strategies. As local search engines, however, mainly only scalarization methods have been utilized (e.g., [2,3,12–14,18]). After a certain euphoria for such methods, this has led to many frustrations as the cost for the resulting hybrid is very high in many cases. The reason for this is the above mentioned missing possibility to fine tune the scalarization methods to obtain a suitable distribution along the solution sets. As a result, scalarization based memetic algorithms are highly competitive in case the given MOP is relatively easy, but this advantage vanishes with increasing complexity of the problem. For complex problems, scalarization based hybrids even perform worse than their base MOEAs due to the relative high cost induced by the local search.

The content of this study is the hybridization of a recently proposed multi-objective continuation method with MOEAs to obtain fast and reliable solvers for continuous MOPs. The Pareto Tracer (PT, [17]) is currently probably the more affordable algorithm (measured regarding the function calls needed to detect the next solution from a given starting point) that allows to perform a solution movement along the Pareto set/front; it is applicable to higher dimensions, and can handle box and equality constraints. In this initial study, we will make the first effort for such hybridization and will restrict ourselves to bi-objective equality constrained problems. We stress, however, that the chosen approach is in general not limited to bi-objective problems. As problems with a more significant



# Sizing CMOS operational transconductance amplifiers applying NSGA-II and MOEAD

<sup>1</sup>Oliver Cuate, <sup>1</sup>Oliver Schuetze, <sup>2</sup>Francesco Grasso, <sup>3</sup>E. Tlelo-Cuautle

<sup>1</sup>CINVESTAV, Department of Computer Science, Mexico City, Mexico. Rodolfo Quintero Chair, UAM Cuajimalpa

<sup>2</sup>Universit di Firenze - Dept. of Information Engineering (DINFO), 50139 Firenze, Italy

<sup>3</sup>Instituto Nacional de Astrofisica, Optica y Electronica (INAOE), Mexico. Email: etlelo@inaoep.mx

**Abstract**—Sizing analog integrated circuits (ICs) is a challenge due to the many trade-offs among their target specifications. For example, it has been shown that the operational transconductance amplifier (OTA) has a huge plethora of applications in analog electronic circuits. It can be designed using complementary metal-oxide-semiconductor (CMOS) IC technology. However, as CMOS technology scales down to the nanometer regime, many design problems arise to accomplish target specifications, and this task become impossible to mitigate when performing traditional manual design. In this manner, this article shows the usefulness of applying multi-objective optimization to size CMOS OTAs. NSGA-II and MOEAD are well-known evolutionary algorithms that can be applied to optimize CMOS ICs, and they are applied herein using CMOS technology of 350 nanometers. Both optimizers are tested using four indicators, namely: hypervolume,  $\Delta_p$ , spacing and coverage. It is highlighted that NSGA-II is better than MOEAD, and it generates feasible solutions providing gains higher than 80dB and bandwidths in the range of MHz.

**Index Terms**—OTA; sizing; optimization; CMOS; NSGA-II, MOEAD

## I. INTRODUCTION

The automatic design of analog circuits is challenging because the IC technology changes every year to accomplish target specifications imposed by modern applications [1]–[3]. In analog circuits, the optimal sizing can reduce the encoding of the design variables [4], because some transistors can have the same sizes and others can have scale values. However, the optimization process will include huge search spaces of the design variables, and the target specifications must accomplish many performances like high-gain and frequency response of an amplifier, wide dynamic range of signal processing, minimal noise and distortion, and so forth. In this manner, some contributions are focused on reaching high frequency responses using silicon-based technology [5]. In the majority of analog designs, complementary metal-oxide-semiconductor (CMOS) IC technology is good enough, and it is useful to accomplish high frequencies [6]. Nanometer CMOS technology requires the estimation of parasitics that generate undesirable effects in analog and RF ICs, and it is not a trivial task as shown in [7]. In this manner, multi-objective optimization approaches have demonstrated to be good methods for the optimal sizing of CMOS ICs [8], and some works include statistical variation analyses [9]. Other contributions introduce hybrid sizing tools [10], which can

be very useful for estimating ranges of voltages and current biases, and of the search spaces of the design variables.

In this article we show the application of two metaheuristics for sizing OTAs, namely: the non-dominated sorting genetic algorithm (NSGA-II [11]) and multi-objective evolutionary algorithm with decomposition (MOEAD [12]). Both metaheuristics are applied to size an OTA using CMOS technology of 350 nanometers, and linking the circuit simulator ngspice [13]. We compare the feasible sized solutions provided by both algorithms using four indicators: hypervolume,  $\Delta_p$ , spacing and coverage. We show the best feasible solutions combining all generations, and also we apply MOEAD adding rules from NSGA-II to handle the constraints on optimizing the OTA. Both metaheuristics have the objectives to maximize gain (measured in dB) and bandwidth (measured in Hz at -3 dB roll-off frequency). In this manner, the sizing is formulated as a bi-objective problem. The constraints accomplish that all MOS transistors work in the saturation mode.

The rest of this article is organized as follows: In Sect. II, NSGA-II and MOEAD are described to perform multi-objective optimization in the sizing of an OTA. Section III details the CMOS OTA and the encoding of the design variables, and formulates the sizing as a bi-objective optimization problem. Numerical results provided by the metaheuristics are given in IV. Finally, the conclusion is given in V.

## II. NSGA-II AND MOEAD

Optimization refers to finding the best possible solution to a problem given a set of constraints [14]. For a multi-objective optimization problem (MOP), one encounters conflicting objectives that provide a set of feasible solutions. In this work, the sizing problem of an OTA is formulated as a bi-objective problem subject to constraints that are associated to the bias conditions of every MOS transistor.

A MOP with  $k$  objectives can be formulated as [15]:

$$\begin{aligned} \min_{x \in \Omega} & F(x), \\ \text{s.t. } & g(x) \leq 0 \\ & h(x) = 0. \end{aligned} \quad (1)$$

In this case,  $F : \Omega \rightarrow \mathbb{R}^k$  is defined as the vector of the objective functions

$$F(x) = (f_1(x), \dots, f_k(x))^T, \quad (2)$$







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## Es Sede UAdeC de NEO Robotics 2019



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La Universidad Autónoma de Coahuila participa como sede del Congreso Neo Robotics Séptimo Taller Internacional sobre Optimización Numérica y Evolutiva 2019, evento que se realiza del 18 al 20 de septiembre en la Sala de Seminarios Emilio J. Talamás en la Unidad Camporredondo.

La actividad reúne a expertos de todos los campos de optimización para discutir, comparar y fusionar perspectivas complementarias, NEO fomenta el desarrollo de métodos híbridos rápidos y confiables que maximicen las fortalezas y minimicen las debilidades de cada paradigma subyacente, a la vez que son aplicables a una clase más amplia de problemas.

La edición de este año de NEO Robotics se realiza en conjunto con el Centro de Investigación y de Estudios Avanzados Cinvestav Saltillo y se centra especialmente en robots móviles y cooperativos, robots de servicio, aplicaciones médicas y agrícolas, técnicas de diseño, control y algoritmos de planificación.

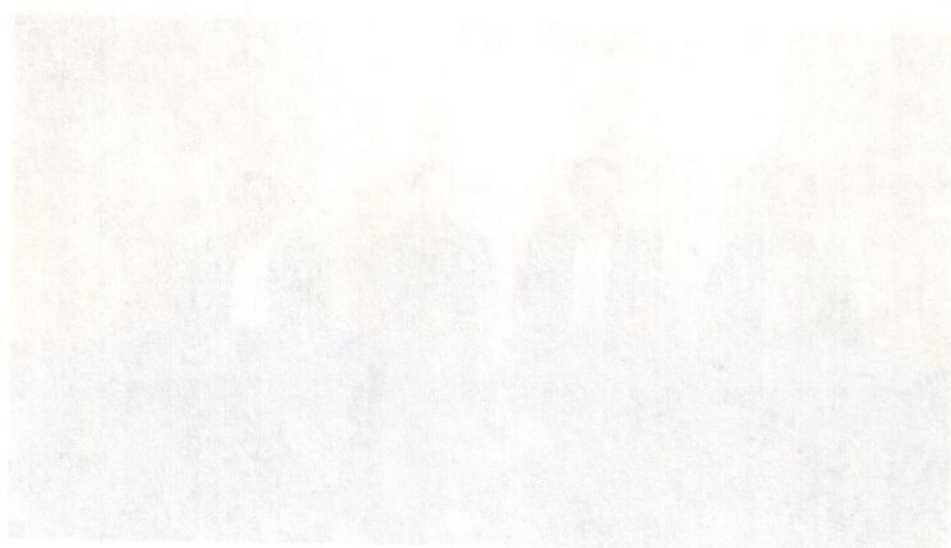
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El presente trabajo tiene como objetivo principal describir el proceso de desarrollo de un robot autónomo capaz de realizar tareas de exploración y mapeo en un entorno desconocido. Para ello, se han seguido los principios de la robótica móvil, la visión por computadora y el aprendizaje automático.

El robot está basado en una plataforma móvil de dos ruedas, equipada con un sensor de ultrasonido para detectar obstáculos y una cámara para capturar imágenes del entorno. El software desarrollado permite al robot navegar por el espacio, evitando obstáculos y creando un mapa del entorno a medida que explora.

Los resultados obtenidos demuestran que el robot es capaz de realizar estas tareas de manera eficiente y autónoma, lo que lo hace una herramienta valiosa para aplicaciones de investigación y desarrollo en robótica móvil.

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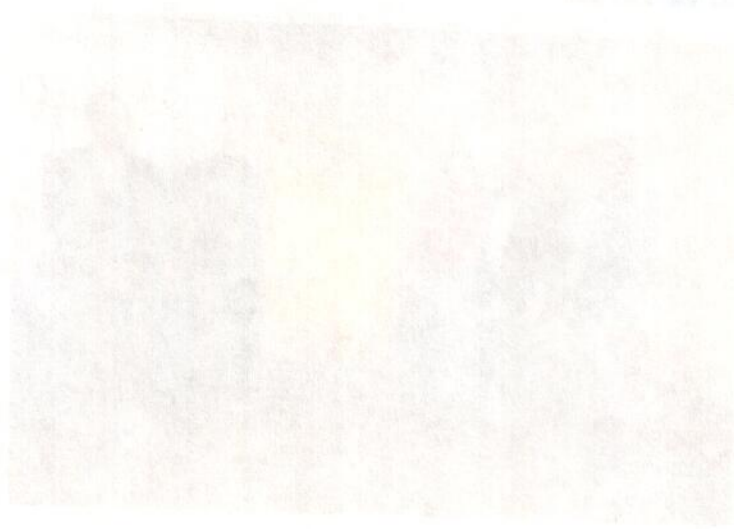
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